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NONLINEAR FUNCTIONAL RANDOM DIFFERENTIAL EQUATION WITH PPF DEPENDENCE

D. S. Palimkar
Department of Mathematics,
Vasantrao Naik College, Nanded

Abstract: In this paper, basic random fixed point theorems with PPF dependence is proved for random operators in separable Banach spaces with different domain and range spaces.

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1. INTRODUCTION

Let R denote the real line and let the closed and bounded interval $I_o = [-r, 0]$ and let $I = [0, T]$ in R , for some real number r > 0 ,T > 0. Let C denote the space of continuous real valued functions defined on I_0 .The space C equipped with the supremum norm $\| \cdot \|_C$ defined by

$$\| \xi \|_C = \sup_{\theta \in I_0} |\xi(\theta)| \quad (1.1)$$

It is clear that C is a Banach space with this norm called the history space of the problem under consideration.For each $t \in I = [0,T]$,define a function $t \to x_t \in C$ by

$$x_t(\theta) = x(t+\theta), \qquad \theta \in I_0 \quad (1.2)$$

Where the argument θ represents the delay in the argument of solutions .Let (Ω, A) be a measurable space .By a mapping $x : \Omega \to C(J, R)$,we denote the function $x(t,\omega)$ which is continuous in the variable t for each $\omega \in \Omega$. In this case case we also wright $x(t,\omega) = x(\omega)(t)$.

Given a measurable function $\phi_0, \phi_1 : \Omega \to C$ and $x : \Omega \to C(I,R)$, consider an initial value problem of second order functional random differential equation of delay type

$$x''(t,\omega) = f\left(t, x_t(\omega), \omega\right)$$

$$x_0(\omega) = \phi_0(\omega) \quad , \quad x_1'(\omega) = \phi_1(\omega)$$

(1.3)

for all $t \in I$ and $\omega \in \Omega$,where $f : I \times C \times \Omega \to R$.

By a random solution x of the random differential equation (1.3) we mean a measurable function $x : \Omega \to C(J,R)$ that satisfies the equation in (1.3) on J ,where $C(J,R)$ is the space of continuous real valued functions defined on $J = I_0 \cup I$.

Here, the existence and uniqueness theorem for FRDE (1.3) are obtained by using random version of classical fixed point theorems of Schauder and Banach respectively .However the of the present paper lies in the nice applicability of our theorem (2.1) for proving the existence of random solution with PPF dependence for the FRDE (1.3) defined on J .

2).AUXILIARY RESULTS

Given a closed and bounded intervals $I = [a,b]$ in R ,the set of real numbers ,for some $a, b \in R,\ a < b$,let $E_0 = C(I,E)$ denote the Banach space of continuous E valued functions defined on I equipped with the supremum norm $\|\ .\ \|_{E_0}$ defined by

$$\|x\|_{E_0} = \sup_{t \in I} \|x(t)\|_E$$

(2.1)

For a fixed $c \in I$,a Razumkhin or minimal class of functions in E_0 is defined as

$$\square_C = \left\{ \phi \in E_0 \middle| \, \|\phi\|_{E_0} = \|\phi(c)\|_E \right\}$$

The class $\square_C$ is algebraically closed with respect to the difference i.e. *if* $\phi - \xi \in \square_C$ whenever $\phi, \xi \in \square_C$. Similarly $\square_C$ is topologically closed if it is closed with respect to the topology on E_0 generated by the norm $\| \cdot \|_{E_0}$. Let $Q : \Omega \times E_0 \to E$ be a random operator. A measurable function $\xi^* : \Omega \to E_0$ is called PPF dependent random fixed point theorem of the random operator $Q(\omega)$ if

$$Q(\omega, \xi^*(\omega)) = \xi^*(c, \omega)$$

for some $c \in I$. Any mathematical statement that guarantees the existence of PPF dependent random fixed point of the random operator $Q(\omega)$ is a random fixed point theorem with PPF dependent or a PPF dependent random fixed point theorem.

The following theorem is used in the study of nonlinear discontinuous random differential equations.

Theorem 2.1(Caratheodory) Let $Q : \Omega \times E_1 \to E_2$ be a mapping such that $Q(\omega, x)$ is measurable in ω for each $x \in E$ and $Q(\omega, x)$ is continuous in x for each $\omega \in \Omega$ .Then the map $(\omega, x) \to Q(\omega, x)$ is jointly measurable.

3. PPF DEPENDENT RANDOM FIXED POINT THEORY

We need the following definitions.

Definition 3.1:A random operator $Q : \Omega \times E_0 \to E$ is called contraction if for each $\omega \in \Omega$

$$\|Q(\omega, \xi) - Q(\omega, \eta)\|_E \le \lambda(\omega) \|\xi - \eta\|_{E_0}$$

(3.1)

For all $\xi, \eta \in E_0$,where $\lambda : \Omega \to R^+$ is a measurable function satisfying $0 \le \lambda(\omega) < 1$ for all $\omega \in \Omega$.

Definition 3.2:A random operator $Q:\Omega\times E_0 \to E$ is called strong random contraction if for a given $c \in I$ and for each $\omega \in \Omega$,

$$\left\|Q(\omega,\xi)-Q(\omega,\eta)\right\|_E \le \lambda(\omega)\left\|\xi(c,\omega)-\eta(c,\omega)\right\|_E$$

(3.2)

For all $\xi,\eta \in E_0$,where $\lambda:\Omega \to R^+$ is measurable function satisfying $0 \le \lambda(\omega) < 1$ for all $\omega \in \Omega$.

Our first random fixed point theorem with PPF dependence is the following result.

Theorem 3.1. Let (Ω, A)be a measurable space and let E be a separable Banach space . If the random operator $Q:\Omega\times E_0 \to E$ is a random contraction, then the following statements holds in E .

(a) If $\square_C$ is algebraically closed with respect to the difference, then for a given $\xi_0 \in E_0$ and for a given $c \in I$, every sequence $\{\xi_n(\omega)\}$ of measurable functions satisfying

$$Q(\omega,\xi_n(\omega)) = \xi_{n+1}(c,\omega)$$

(3.3)

and

$$\left\|\xi_n(\omega)-\xi_{n+1}(\omega)\right\|_{E_0} = \left\|\xi_n(c,\omega)-\xi_{n+1}(c,\omega)\right\|_E$$

(3.4)

Converges to a PPF dependent random fixed point of the random operator $Q(\omega)$,i.e. there is a measurable function $\xi^*:\Omega \to E_0$ such that for each $\omega \in \Omega$,

$$Q(\omega,\xi^*(\omega)) = Q(\omega)\xi^*(\omega) = \xi^*(c,\omega)$$

(b).Given $\xi_0,\eta_0 \in E_0$,let$\{\xi_n(\omega)\}$ *and* $\{\eta_n(\omega)\}$ be the sequences of iterates of measurable functions corresponding to ξ_0 and η_0 constructed as in (a).Then ,

$$\|\xi_n(\omega)-\eta_n(\omega)\|_{E_0} \le \frac{1}{1-\lambda(\omega)}\left[\|\xi_0-\xi_1(\omega)\|_{E_0}+\|\xi_0-\xi_1(\omega)\|_{E_0}\right]+\|\xi_0-\eta_0\|_{E_0}$$

If ,in particular, $\xi_0=\eta_0$ and $\{\xi_n(\omega)\}\neq\{\eta_n(\omega)\}$,then

$$\|\xi_n(\omega)-\eta_n(\omega)\|_{E_0} \le \frac{2}{1-\lambda(\omega)}\|\xi_0-\xi_1(\omega)\|_{E_0}.$$

(c).Finally if $\mathcal{R}_C$ is topologically closed, then for a given $\xi_0\in E_0$, every sequence $\{\xi_n(\omega)\}$ of iterates of $Q(\omega)$ constructed as in (a), converges to a unique PPF dependent random random fixed point $\xi^*(\omega)$ of $Q(\omega)$ i.e. there is a unique measurable function $\xi^*:\Omega\to E_0$ such that

$Q(\omega,\xi^*(\omega))=\xi^*(c,\omega)$ for all $\omega\in\Omega$.

Theorem3.2 :[Hybrid fixed point theorem with PPF dependence]

Let (Ω,A) be a measurable space and let E be a separable banach space. Suppose that $A:\Omega\times E_0\to E$ and $\mathrm{B}:\Omega\times E\to E$ are two continuous random operators satisfying for each $\omega\in\Omega$,

a) $A(\omega)$ is a strong random contraction, and

b) B is compact on $\Omega\times E$.

If $\mathcal{R}_C$ is algebraically and topologically closed with respect to the difference, then for a given $c\in I$, the random operator equation

$$A(\omega,\xi(\omega))+B(\omega,\xi(\omega,c))=\xi(\omega,c)$$
(3.5)

has a random solution with PPF dependence i.e. for given $c\in I$, there is a measurable function $\xi^*:\Omega\to E_0$ such that

$A(\omega,\xi^*(\omega))+B(\omega,\xi^*(c,\omega))=\xi^*(c,\omega)$ for all $\omega\in\Omega$.

4.MAIN RESULT

We consider the following hypotheses.

(H_1) The function $\omega\to f(t,x,\omega)$ is measurable for each $t\in I$ and $x\in C$ and

the function $(t,x)\to f(t,x,\omega)$ is jointly continuous for each $\omega\in\Omega$.

(H_2) There exist a real number $M_f>0$ such that for each $\omega\in\Omega$,

$$|f(t,x,\omega)|\le M_f\,, \quad \text{for all } t\in I \text{ and } x\in C$$

(H_3)There exist a real number $L>0$ such that for each $\omega\in\Omega$,

$$|f(t,x,\omega)-f(t,y,\omega)|\le L\|x-y\|_C\,, \quad \text{for all } t\in I \text{ and } x,y\in C$$

(H_4)The function $\phi_0,\phi_1:\Omega\to C$ are measurable and bounded with

$$Q_0=\sup_{\omega\in\Omega}\phi_0(\omega),\ Q_1=\sup_{\omega\in\Omega}\phi_1(\omega)$$

Theorem 4.1 . Assume that the hypothesis (H_1) through (H_4) holds. Furthermore , if $LT<1$, then the $FRDE$ (1.3) has a unique PPF dependent random solution defined on J .

Proof.Set $E=C(J,R)$. Then E is a Banach space with respect to the usual supremum norm

$$\|\,.\,\|_E=\sup_{t\in J}|x(t)|$$

(4.1)

Clearly E is a separable Banach space. Given a function $x\in C(J,R)$, define a mapping $\hat{x}:I\to C$ by $\hat{x}(t)=x_t\in C$ so that $\hat{x}(t)(0)=x_t(0)=x(t),\ t\in I$ *and* $\hat{x}(0)=x_0$.

Define a set $\hat{E}$ of functions by

$$\hat{E}=\left\{\hat{x}=\left(x_t\right)_{t\in I}: x_t\in C, x\in C\left(I,R\right) \; and \; x_0=\phi\right\}$$

(4.2)

Define a norm $\|\hat{x}\|_{\hat{E}}$ in $\hat{E}$ by

$$\|\hat{x}\|_{\hat{E}}=\sup_{t\in I}\|x_t\|_C$$

(4.3)

Clearly, $\hat{x}\in C\left(I_0,R\right)=C$. Next we show that $\hat{E}$ is a Banach space. Consider a Cauchy sequence $\{\hat{x}_n\}$ in $\hat{E}$. For simplicity of notations ,we denote $\hat{x}_n(t)=x_t^n$. Then $\left\{\left(x_t^n\right)_{t\in I}\right\}$ is a Cauchy sequence in C for each $t\in I$. This further implies that $\{x_t^m(s)\}$ is a Cauchy sequence in R for each $s\in[-r,0]$. Then $\{x_t^m(s)\}$ converges to $x_t(s)$ for each $t\in I_0$. Since sequence $\{x_t^n\}$ is a sequence of uniformly continuous functions for each $t\in I$, $x_t(s)$ is also continuous in $s\in[-r,0]$. Hence the sequence $\{\hat{x}_n\}$ converges to $\hat{x}\in\hat{E}$. As a result, $\hat{E}$ is complete. Moreover, $\hat{E}$ is a separable Banachspace.

Now the FRDE (1.3) is equivalent to the nonlinear functional random integral equation

(in short FRIE)

$$x(t,\omega)=\begin{cases}\phi_0(\omega)+\phi_1(\omega)t+\int_0^t(t-s)\,f\left(s,x_s(\omega),\omega\right)ds & if\ t\in I\\ \phi_0(t,\omega) & if\ t\in I_0\end{cases}$$

(4.4)

Given a measurable function $\hat{x}:\Omega\to\hat{E}$, consider the operator $Q:\Omega\times\hat{E}\to$ □ defined by

$$Q\left(\omega,\hat{x}(\omega)\right)=Q\left(\omega,x_t(\omega)\right)$$

$$= \begin{cases} \phi_0(\omega) + \phi_1(\omega)t + \int_0^t (t-s) f(s, x_s(\omega), \omega) ds & \text{if } t \in I \\ \phi_0(t, \omega) & \text{if } t \in I_0 \end{cases}$$

(4.5)

Then FRDE (4.4)is equivalent to the random operator equation

$Q(\omega, \hat{x}(\omega)) = \hat{x}(0, \omega) = \hat{x}(\omega)(0)$ (4.6)

Define a sequence $\{\hat{x}_n(\omega)\}$ of measurable functions by

$$\left.\begin{array}{l} (i)\ Q(\omega, \hat{x}_n(\omega)) = \hat{x}_{n+1}(\omega)(0) \\ (ii)\ \| \hat{x}_n(\omega) - \hat{x}_{n+1}(\omega) \|_{E_0} = \| \hat{x}_n(\omega)(0) - \hat{x}_{n+1}(\omega)(0) \|_E \end{array}\right\} \quad (4.7)$$

For $n = 1, 2, 3, ...$

We shall show that the operator Q sarisfies condition (a) of theorem (3.1)on $\Omega \times \hat{E}$. Firstly, we show that Q is a random operator on $\Omega \times \hat{E}$. Since the hypothesis (H_1) holds, by Caratheodory theorem, the function $\omega \to f(t, x, \omega)$ is measurable for all $t \in I$ and $x \in C$. As the integral is the limit of finite sum of measurable functions, so the map

$$\omega \to \int_0^t (t-s) f(s, x_s(\omega), \omega) ds$$

Is measurable. Again the sum of two measurable function is again measurable, so the map

$$\omega \to \phi_0(\omega) + \phi_1(\omega)t + \int_0^t (t-s) f(s, x_s(\omega), \omega) ds$$

Is measurable. Hence, the operator $Q(\omega, \hat{x})$ is measurable in ω for each $\hat{x} \in \hat{E}$. As a results $Q(\omega)$ is a random operator on $\hat{E}$ into E.

Secondly, we show that random operator $Q(\omega)$ is continuous on $\hat{E}$. Let $\omega \in \Omega$ be fixed. We show that the continuity of the random operator $Q(\omega)$ in following two cases

Case I : Let $t \in [0, \mathrm{T}]$ and let $\{\hat{x}_n(\omega)\}$ be a sequence of points in $\hat{E}$ such that $\hat{x}_n(\omega) \to \hat{x}(\omega)$ as $n \to \infty$. Then by dominated convergence theorem,

$$\lim_{n\to\infty} Q(\omega, \hat{x}_n(t,\omega)) = \lim_{n\to\infty}\left(\phi_0(\omega) + \phi_1(\omega)t + \int_0^t (t-s)\, f\left(s, x_s^n(\omega), \omega\right) ds\right)$$

$$= \phi_0(\omega) + \phi_1(\omega)t + \lim_{n\to\infty}\left(\int_0^t (t-s)\, f\left(s, x_s^n(\omega), \omega\right) ds\right)$$

$$= \phi_0(\omega) + \phi_1(\omega)t + \left(\int_0^t (t-s) \lim_{n\to\infty} f\left(s, x_s^n(\omega), \omega\right) ds\right)$$

$$= \phi_0(\omega) + \phi_1(\omega)t + \int_0^t (t-s)\, f\left(s, x_s(\omega), \omega\right) ds$$

$$= Q(\omega, \hat{x}(t,\omega))$$

For all $t \in [0, T]$ and for each fixed $\omega \in \Omega$.

Case II : suppose that $t \in [-r, 0]$. Then we have,

$$\left|Q(\omega, \hat{x}_n(\omega)) - Q(\omega, \hat{x}(\omega))\right| = \left|\phi_0(t,\omega) - \phi_0(t,\omega)\right| = 0$$

For each fixed $\omega \in \Omega$. Hence,

$$\lim_{n\to\infty} Q(\omega, \hat{x}_n(t,\omega)) = Q(\omega, \hat{x}(t,\omega))$$

For all $t \in [-r, 0]$ and $\omega \in \Omega$. Now combining the case I and II, we conclude that $Q(\omega)$ is a pointwise continuous random operator on $\hat{E}$ into itself.

Now we show that the family of functions $\{Q(\omega, \hat{x}_n(\omega))\}$ is uniformly continuous set in E for a fixed $\omega \in \Omega$. We consider the following the following three cases :

Case 1: let $\varepsilon > 0$ and let $t_1, t_2 \in [0, T]$ be arbitrary. Then we have

$$\left| Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right) \right|$$

$$\leq \left| \phi_0(\omega) + \phi_1(\omega) t_1 + \int_0^{t_1} (t_1 - s) f\left(s, x_s^n(\omega), \omega\right) - \phi_0(\omega) - \phi_1(\omega) t_2 - \int_0^{t_2} (t_2 - s) f\left(s, x_s^n(\omega), \omega\right) \right|$$

$$\leq \left|\phi_1(\omega)\right| \left|t_1 - t_2\right| + \left| \int_0^{t_1} (t_1 - s) f\left(s, x_s^n(\omega), \omega\right) - \int_0^{t_2} (t_2 - s) f\left(s, x_s^n(\omega), \omega\right) \right|$$

$$\leq \left|\phi_1(\omega)\right| \left|t_1 - t_2\right| + \left| \int_{t_2}^{t_1} (t_1 + t_2 - 2s) f\left(s, x_s^n(\omega), \omega\right) \right|$$

$$\leq \left|\phi_1(\omega)\right| \left|t_1 - t_2\right| + \int_{t_2}^{t_1} \left|(t_1 + t_2 - 2s)\right| \left| f\left(s, x_s^n(\omega), \omega\right) \right| ds$$

$$\leq Q_1 \left|t_1 - t_2\right| + M_f K \left|t_1 - t_2\right|$$

$$\leq \left(Q_1 + M_f K\right) \left|t_1 - t_2\right|$$

Choose $\delta_1 = \dfrac{\varepsilon}{2\left(Q_1 + M_f K + 1\right)} > 0$. Then, if $\left|t_1 - t_2\right| < \delta_1$ implies

$$\left| Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right) \right| < \frac{\left(Q_1 + M_f K\right)\varepsilon}{2\left(Q_1 + M_f K + 1\right)}$$

Uniformly for $x_t^n = \hat{x}_n \in E_0$.

Case II : Let $t_1, t_2 \in [-r, 0]$ be arbitrary. Since $t \to \phi_0(\omega, t)$, is continuous on a compact $[-r, 0]$, it is uniformly continuous there. Hence, for above $\varepsilon > 0$ there exist a $\delta > 0$ such that $\left|t_1 - t_2\right| < \delta_1$ implies

$$\left| Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right) \right| = \left| \phi_0(t_1, \omega) - \phi_0(t_2, \omega) \right| \leq \frac{\varepsilon}{2\left(Q_1 + M_f K + 1\right)}$$

Uniformly for $\hat{x}_n \in E_0$.

Case III : Let $t_1 \in [-r,0]$ and $t_2 \in [0,\mathrm{T}]$ be arbitrary. Choose $\delta = \min(\delta_1,\delta_2)$.

Then, $|t_1 - t_2| < \delta$ implies

$$\left|Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right)\right| \le \left|Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_0}^n(\omega)\right)\right| + \left|Q\left(\omega, x_{t_0}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right)\right|$$

$$< \frac{(Q_1 + M_f K)\varepsilon}{2(Q_1 + M_f K + 1)} + \frac{\varepsilon}{2(Q_1 + M_f K + 1)}$$

$$< \varepsilon$$

Uniformly for $\hat{x}_n \in E_0$.

Thus in all three cases, $|t_1 - t_2| < \delta$ implies

$$\left|Q\left(\omega, x_{t_1}^n(\omega)\right) - Q\left(\omega, x_{t_2}^n(\omega)\right)\right| < \varepsilon$$

Uniformly for all $t_1, t_2 \in J$ and $\hat{x}_n \in E_0$. This shows that $\{Q(\omega, \hat{x}_n)\}$ is a sequence of uniformly continuous functions on J. Hence, it converges uniformly on J. Hence, $Q(\omega, \hat{x})$ is a continuous random operator on $\hat{E}$ for a fixed $\omega \in \Omega$.

Finally we show that Q is a random contraction on $\hat{E}$. Let $\omega \in \Omega$ be fixed. Then,

$$\left\|Q(\omega, \hat{x}(\omega)) - Q(\omega, \hat{y}(\omega))\right\|_E = \left\|Q(\omega, x_t(\omega)) - Q(\omega, y_t(\omega))\right\|_E$$

$$\le \sup_{t \in I} \left| \int_0^t (t-s)\, f(s, x_s(\omega), \omega) - \int_0^t (t-s)\, f(s, y_s(\omega), \omega) \right|$$

$$\le \int_0^T L \left\| x_s(\omega) - y_s(\omega) \right\|_C ds$$

$$\le \int_0^T L \left\| \hat{x}(\omega) - \hat{y}(\omega) \right\|_{\hat{E}} ds$$

$$\leq LT\left\|\hat{x}(\omega)-\hat{y}(\omega)\right\|_{\hat{E}}$$

(4.8)

For all $\hat{x}(\omega),\hat{y}(\omega)\in\hat{E}$. Hence Q is s random contraction on $\hat{E}$ with contraction constant

$\alpha = LT < 1$.

Thus the condition (a)of theorem (3.1) is satisfied. Hence, an application of theorem 3.1(a) yields that the functional random integral equation (4.6) has a random solution with PPF dependence defined on J. This further implies that the FRDE (1.3) has a PPF dependence random solution ξ^* defined on J and the sequence $\{\xi_n(\omega)\}$ of measurable functions constructed as in (4.7) converges to ξ^*. Moreover, here the Razumikhin class $\square_0$, $0\in[-r,T]$ is $C([0,T],R)$ which is algebraically and topologically closed with respect to difference, then by theorem 3.1(c), ξ^* is a unique random solution with PPF dependence for the FRDE (1.3)defined on J. This completes the proof.

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